

June, 2016

Revised version: September, 2016

Higher spin super-Cotton tensors and generalisations of the linear-chiral duality in three dimensions

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Abstract

In three spacetime dimensions, (super)conformal geometry is controlled by the (super) Cotton tensor. We present a new duality transformation for \mathcal{N} -extended supersymmetric theories formulated in terms of the linearised super-Cotton tensor or its higher spin extensions for the cases $\mathcal{N} = 2, 1, 0$. In the $\mathcal{N} = 2$ case, this transformation is a generalisation of the linear-chiral duality, which is known to provide a dual description in terms of chiral superfields for general models of self-interacting $\mathcal{N} = 2$ vector multiplets in three dimensions and $\mathcal{N} = 1$ tensor multiplets in four dimensions. For superspin-1 (gravitino multiplet), superspin-3/2 (supergravity multiplet) and higher superspins $s \geq 2$, the duality transformation relates a higher-derivative theory to one containing at most two derivatives at the component level. In the $\mathcal{N} = 1$ case, we introduce gauge prepotentials for higher spin superconformal gravity and construct the corresponding super-Cotton tensors, as well as the higher spin extensions of the linearised $\mathcal{N} = 1$ conformal supergravity action. Our $\mathcal{N} = 1$ duality transformation is a higher spin extension of the known superfield duality relating the massless $\mathcal{N} = 1$ vector and scalar multiplets. In the non-supersymmetric ($\mathcal{N} = 0$) case, the gauge prepotentials for higher spin conformal geometry (both bosonic and fermionic) and the corresponding Cotton tensors can be obtained from their $\mathcal{N} = 1$ counterparts by carrying out $\mathcal{N} = 1 \rightarrow \mathcal{N} = 0$ reduction. In the bosonic sector, this reduction is shown to lead to the Cotton tensors for higher spins constructed by Pope and Townsend, and by Damour and Deser in the spin-3 case. Our $\mathcal{N} = 0$ duality transformation is a higher spin extension of the vector-scalar duality.

1 Introduction

Recently there has been renewed interest in dualities in three-dimensional (3D) field theories [1, 2, 3]. In this paper we consider 3D duality transformations for theories involving higher spin analogues of the Cotton tensor or its supersymmetric generalisations – the $\mathcal{N} = 1$ and $\mathcal{N} = 2$ super-Cotton tensors [4, 5, 6]. The specific feature of 3D conformal gravity is that its geometry can be formulated such that the Cotton tensor fully determines the algebra of covariant derivatives, see e.g. [6]. Similarly, in 3D \mathcal{N} -extended conformal supergravity formulated in conformal superspace [6], the corresponding superspace geometry is controlled by the super-Cotton tensor. In the $\mathcal{N} = 2$ case, our higher spin duality transformation may be thought of as a generalisation of the famous linear-chiral duality.

The linear-chiral duality [7, 8] is of fundamental importance in supersymmetric field theory, supergravity and string theory, in particular in the context of supersymmetric non-linear sigma models [8, 9, 10]. It provides a dual description in terms of chiral superfields for general models of self-interacting 3D $\mathcal{N} = 2$ vector multiplets or 4D $\mathcal{N} = 1$ tensor multiplets [7]. The only assumption for the duality to work is that the 3D $\mathcal{N} = 2$ vector multiplet or 4D $\mathcal{N} = 1$ tensor multiplet appears in the superfield Lagrangian, $L(W)$, only via its field strength W , which is a real linear superfield,

$$\bar{D}^2 W = 0, \quad W = \bar{W} \implies D^2 W = 0. \quad (1.1)$$

It is pertinent to recall the definition of the linear-chiral duality in the 3D $\mathcal{N} = 2$ case we are interested in. We start from a self-interacting vector multiplet model with action

$$S[W] = \int d^3x d^2\theta d^2\bar{\theta} L(W), \quad (1.2)$$

and associate with it the following first-order model

$$S[\mathcal{W}, \Psi, \bar{\Psi}] = \int d^3x d^2\theta d^2\bar{\theta} \left\{ L(\mathcal{W}) - (\Psi + \bar{\Psi})\mathcal{W} \right\}, \quad \bar{D}_\alpha \Psi = 0. \quad (1.3)$$

Here the dynamical variables are a real unconstrained superfield \mathcal{W} , a chiral scalar Ψ and its complex conjugate $\bar{\Psi}$. Varying $S[\mathcal{W}, \Psi, \bar{\Psi}]$ with respect to the Lagrange multiplier Ψ gives the equation of motion $\bar{D}^2 \mathcal{W} = 0$, and hence $\mathcal{W} = W$. Then the second term on the right of $S[\mathcal{W}, \Psi, \bar{\Psi}]$ drops out, and we are back to the vector multiplet model (1.2). On the other hand, we can vary (1.3) with respect to \mathcal{W} resulting in the equation of motion

$$L'(\mathcal{W}) = \Psi + \bar{\Psi}. \quad (1.4)$$

This equation allows us to express \mathcal{W} as a function of Ψ and $\bar{\Psi}$, and then (1.3) turns into the dual action

$$S_D[\Psi, \bar{\Psi}] = \int d^3x d^2\theta d^2\bar{\theta} L_D(\Psi, \bar{\Psi}) . \quad (1.5)$$

It remains to point out that the constraint (1.1) is solved in the 3D case by

$$W = \Delta H , \quad \Delta = \frac{i}{2} D^\alpha \bar{D}_\alpha , \quad (1.6)$$

where the real prepotential H is defined modulo gauge transformations of the form

$$\delta H = \lambda + \bar{\lambda} , \quad \bar{D}_\alpha \lambda = 0 . \quad (1.7)$$

It is worth recalling one more type of duality that is naturally defined in the 3D $\mathcal{N} = 1$ and 4D $\mathcal{N} = 2$ cases, the so-called complex linear-chiral duality [11, 12] (see also [13] for a review). It provides a dual description in terms of chiral superfields for general models of self-interacting complex linear superfields Γ and their conjugates $\bar{\Gamma}$. The complex linear-chiral duality plays a fundamental role in the context of off-shell supersymmetric sigma models with eight supercharges [10, 14]. The complex linear superfield Γ is defined by the only constraint

$$\bar{D}^2 \Gamma = 0 . \quad (1.8)$$

The complex linear-chiral duality works as follows. Consider a 3D $\mathcal{N} = 2$ supersymmetric field with action

$$S[\Gamma, \bar{\Gamma}] = \int d^3x d^2\theta d^2\bar{\theta} L(\Gamma, \bar{\Gamma}) . \quad (1.9)$$

We associate with it a first-order action of the form

$$S[V, \bar{V}, \Psi, \bar{\Psi}] = \int d^3x d^2\theta d^2\bar{\theta} \left\{ L(V, \bar{V}) - \Psi V - \bar{\Psi} \bar{V} \right\} , \quad \bar{D}_\alpha \Psi = 0 . \quad (1.10)$$

Here the dynamical superfields comprise a complex unconstrained scalar V , a chiral scalar Ψ and their conjugates. Varying (1.10) with respect to the Lagrange multiplier Ψ gives $V = \Gamma$, and then the second term in (1.10) drops out as a consequence of the identities

$$\int d^3x d^2\theta d^2\bar{\theta} U = -\frac{1}{4} \int d^3x d^2\theta \bar{D}^2 U = -\frac{1}{4} \int d^3x d^2\bar{\theta} D^2 U , \quad (1.11)$$

for any superfield U . As a result, the first-order action reduces to the original one, $S[\Gamma, \bar{\Gamma}]$. On the other hand, we can consider the equation of motion for V ,

$$\frac{\partial}{\partial V} L(V, \bar{V}) = \Psi , \quad (1.12)$$

and its conjugate. The latter equations allow us to express the auxiliary superfields V and \bar{V} in terms of Ψ and $\bar{\Psi}$. Then (1.10) turns into the dual action

$$S_D[\Psi, \bar{\Psi}] = \int d^3x d^2\theta d^2\bar{\theta} L_D(\Psi, \bar{\Psi}) . \quad (1.13)$$

It should be mentioned that the complex linear-chiral duality can also be introduced in the reverse order, by starting with a chiral model

$$S[\Psi, \bar{\Psi}] = \int d^3x d^2\theta d^2\bar{\theta} K(\Psi, \bar{\Psi}) , \quad \bar{D}_\alpha \Psi = 0 , \quad (1.14)$$

and then applying a Legendre transformation to $S[\Psi, \bar{\Psi}]$ in order to result in a model described by a complex linear superfield Γ and its conjugate $\bar{\Gamma}$. This makes use of the first-order action

$$S[U, \bar{U}, \Gamma, \bar{\Gamma}] = \int d^3x d^2\theta d^2\bar{\theta} \left\{ K(U, \bar{U}) - \Gamma U - \bar{\Gamma} \bar{U} \right\} , \quad \bar{D}^2 \Gamma = 0 . \quad (1.15)$$

In the 4D $\mathcal{N} = 1$ case, the first-order action (1.15) with $K(U, \bar{U}) = U\bar{U}$ was considered for the first time by Zumino [20]. However, he did not realise the fact that this construction leads to a new off-shell description for the scalar multiplet, which was an observation made in [11, 12].

The higher-spin generalisation of the complex linear-chiral duality has been given in [15, 16, 17, 19]. For every integer $2s = 3, 4, \dots$, it relates the two off-shell formulations for the massless superspin- s multiplet in four dimensions constructed¹ in [15, 16, 17] and for the massive superspin- s multiplet in three dimensions presented in [19]. The goal of this paper is to give higher-spin generalisations of the 3D $\mathcal{N} = 2$ linear-chiral duality in three dimensions and its $\mathcal{N} = 1$ and $\mathcal{N} = 0$ cousins.

This paper is organised as follows. The $\mathcal{N} = 2$ duality transformation is presented in section 2. In section 3 we introduce gauge prepotentials for higher spin superconformal geometry, construct the corresponding super-Cotton tensors, and present the higher spin extension of the linearised $\mathcal{N} = 1$ conformal supergravity action. Our $\mathcal{N} = 1$ duality transformation is also described in this section. Finally, section 4 is devoted to the non-supersymmetric ($\mathcal{N} = 0$) higher spin duality transformation.

¹See [18] for a review of the models proposed in [15, 16].

2 $\mathcal{N} = 2$ duality

Let n be a positive integer. We recall the higher-spin $\mathcal{N} = 2$ superconformal field strength, $W_{\alpha(n)} = \bar{W}_{\alpha(n)}$, introduced in [19]

$$W_{\alpha_1 \dots \alpha_n}(H) := \frac{1}{2^{n-1}} \sum_{J=0}^{\lfloor n/2 \rfloor} \left\{ \binom{n}{2J} \Delta^{\square^J} \partial_{(\alpha_1}^{\beta_1} \dots \partial_{\alpha_{n-2J}}^{\beta_{n-2J}} H_{\alpha_{n-2J+1} \dots \alpha_n) \beta_1 \dots \beta_{n-2J}} \right. \\ \left. + \binom{n}{2J+1} \Delta^2 \square^J \partial_{(\alpha_1}^{\beta_1} \dots \partial_{\alpha_{n-2J-1}}^{\beta_{n-2J-1}} H_{\alpha_{n-2J} \dots \alpha_n) \beta_1 \dots \beta_{n-2J-1}} \right\}, \quad (2.1)$$

where $\lfloor x \rfloor$ denotes the floor (or the integer part) of a number x . The field strength $W_{\alpha(n)}$ is a descendant of the real unconstrained prepotential $H_{\alpha(n)}$ defined modulo gauge transformations of the form

$$\delta H_{\alpha(n)} = g_{\alpha(n)} + \bar{g}_{\alpha(n)}, \quad g_{\alpha_1 \dots \alpha_n} = \bar{D}_{(\alpha_1} L_{\alpha_2 \dots \alpha_n)}, \quad (2.2a)$$

where the complex gauge parameter $g_{\alpha(n)}$ is an arbitrary longitudinal linear superfield,

$$\bar{D}_{(\alpha_1} g_{\alpha_2 \dots \alpha_{n+1})} = 0. \quad (2.2b)$$

The field strength is invariant under the gauge transformations (2.2),

$$\delta W_{\alpha(n)} = 0, \quad (2.3)$$

and obeys the Bianchi identity

$$\bar{D}^\beta W_{\beta \alpha_1 \dots \alpha_{n-1}} = 0 \iff D^\beta W_{\beta \alpha_1 \dots \alpha_{n-1}} = 0, \quad (2.4)$$

which implies

$$\bar{D}^2 W_{\alpha(n)} = D^2 W_{\alpha(n)} = 0. \quad (2.5)$$

As demonstrated in [19], $W_{\alpha(n)}$ is a primary superfield of dimension $(1 + n/2)$ if the prepotential $H_{\alpha(n)}$ is chosen to be primary of dimension $(-n/2)$. Associated with $W_{\alpha(n)}$ is the $\mathcal{N} = 2$ superconformal Chern-Simons action [19]

$$S_{\text{CS}}[H] = i^n \int d^3x d^2\theta d^2\bar{\theta} H^{\alpha(n)} W_{\alpha(n)}(H), \quad (2.6)$$

which is invariant under the gauge transformations (2.2). In the $n = 0$ case, the Bianchi identity (2.4) should be replaced with (1.1).

In the $n = 2$ case, the field strength $W_{\alpha\beta}(H)$ coincides with the linearised version [21, 19] of the $\mathcal{N} = 2$ super-Cotton tensor [4, 6]. Thus the field strength (2.1) for $n > 2$ is the higher-spin extension of the super-Cotton tensor.

For $n = 0$ the Bianchi identity (2.4) is not defined, and is instead replaced with its corollary (2.5). In this case, the expression (2.1) reduces to the vector multiplet field strength (1.6), and the gauge invariance (2.2) turns into (1.7). Finally, for $n = 0$ the action (2.6) reduces to the topological mass term for the Abelian vector multiplet [22, 23, 24].

Of crucial importance for our analysis is the fact that the expression (2.1) is the general solution to the constraint (2.4). This observation is the key to introducing a new type of duality. Let us consider a higher-derivative $\mathcal{N} = 2$ superconformal theory with action

$$S[W] = \int d^3x d^2\theta d^2\bar{\theta} \Phi \bar{\Phi} L \left(\frac{W_{\alpha(n)}}{(\Phi \bar{\Phi})^{1+n/2}} \right) , \quad (2.7)$$

where the superconformal compensator Φ is a nowhere vanishing primary superfield of dimension $1/2$. The origin of Φ is not important for us. In particular, Φ may be frozen to a constant value, and then we result with a theory described solely in terms of the higher-spin gauge superfield $H_{\alpha(n)}$. We can associate with (2.7) the following first-order model

$$S[\mathcal{W}, G, \bar{G}] = \int d^3x d^2\theta d^2\bar{\theta} \left\{ \Phi \bar{\Phi} L \left(\frac{\mathcal{W}_{\alpha(n)}}{(\Phi \bar{\Phi})^{1+n/2}} \right) - i^n (G^{\alpha(n)} + \bar{G}^{\alpha(n)}) \mathcal{W}_{\alpha(n)} \right\} , \quad (2.8)$$

where $\mathcal{W}_{\alpha(n)}$ is a real unconstrained superfield, while $G_{\alpha(n)}$ is a longitudinal linear superfield,

$$\bar{D}_{(\alpha_1} G_{\alpha_2 \dots \alpha_{n+1})} = 0 . \quad (2.9)$$

The general solution of this constraint is

$$G_{\alpha_1 \dots \alpha_n} = \bar{D}_{(\alpha_1} \zeta_{\alpha_2 \dots \alpha_n)} , \quad (2.10)$$

for some unconstrained complex prepotential $\zeta_{\alpha(n-1)}$. Varying (2.8) with respect to $G^{\alpha(n)}$ and its conjugate gives

$$\bar{D}^\beta \mathcal{W}_{\beta \alpha_1 \dots \alpha_{n-1}} = 0 \quad \Longleftrightarrow \quad D^\beta \mathcal{W}_{\beta \alpha_1 \dots \alpha_{n-1}} = 0 , \quad (2.11)$$

which means that $\mathcal{W}_{\alpha(n)} = W_{\alpha(n)}$. Plugging this in $S[\mathcal{W}, G, \bar{G}]$, the second term in (2.8) drops out, and we return to the original action (2.7). On the other hand, we can start

from the first-order model (2.8) and integrate out the auxiliary superfield $\mathcal{W}_{\alpha(n)}$. This leads to a dual action of the form

$$S_D[G, \bar{G}] = \int d^3x d^2\theta d^2\bar{\theta} \Phi \bar{\Phi} L_D \left((G^{\alpha(n)} + \bar{G}^{\alpha(n)}) (\Phi \bar{\Phi})^{n/2} \right). \quad (2.12)$$

Unlike the original action (2.7), its dual $S_D[G, \bar{G}]$ does not contain higher derivatives.

In describing our duality transformation, we assumed $n > 0$. It is easy to see that it reduces to the linear-chiral duality in the $n = 0$ case.

3 $\mathcal{N} = 1$ duality

The results in the previous section can be used to obtain a new type of $\mathcal{N} = 1$ duality by carrying out the $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ superspace reduction sketched in [19]. Applying this reduction to (2.1) leads to the higher-spin $\mathcal{N} = 1$ superconformal field strength²

$$W_{\alpha_1 \dots \alpha_n}(H) := \frac{1}{2^{n-1}} \sum_{J=0}^{\lfloor n/2 \rfloor} \left\{ \binom{n}{2J} \square^J \partial_{(\alpha_1}^{\beta_1} \dots \partial_{\alpha_{n-2J}}^{\beta_{n-2J}} H_{\alpha_{n-2J+1} \dots \alpha_n) \beta_1 \dots \beta_{n-2J}} \right. \\ \left. - \frac{i}{2} \binom{n}{2J+1} D^2 \square^J \partial_{(\alpha_1}^{\beta_1} \dots \partial_{\alpha_{n-2J-1}}^{\beta_{n-2J-1}} H_{\alpha_{n-2J} \dots \alpha_n) \beta_1 \dots \beta_{n-2J-1}} \right\}, \quad (3.1)$$

which is real, $W_{\alpha(n)} = \bar{W}_{\alpha(n)}$. The field strength $W_{\alpha(n)}$ is a descendant of the real unconstrained prepotential $H_{\alpha(n)}$ defined modulo gauge transformations of the form

$$\delta H_{\alpha(n)} = i^n D_{(\alpha_1} \zeta_{\alpha_2 \dots \alpha_n)}, \quad \bar{\zeta}_{\alpha(n-1)} = \zeta_{\alpha(n-1)}. \quad (3.2)$$

The field strength is invariant under the gauge transformations (3.2),

$$\delta W_{\alpha(n)} = 0, \quad (3.3)$$

and obeys the Bianchi identity

$$D^\beta W_{\beta \alpha_1 \dots \alpha_{n-1}} = 0. \quad (3.4)$$

It may be shown that $W_{\alpha(n)}$ is a primary superfield of dimension $(1 + n/2)$, in the sense of [26, 27], if the prepotential $H_{\alpha(n)}$ is primary of dimension $(1 - n/2)$. Associated with $W_{\alpha(n)}$ is the $\mathcal{N} = 1$ superconformal Chern-Simons action

$$S_{CS}[H] = i^{n-1} \int d^3x d^2\theta H^{\alpha(n)} W_{\alpha(n)}(H), \quad (3.5)$$

²See [25] for a detailed derivation.

which is invariant under the gauge transformations (3.2). The superconformal invariance of $S_{\text{CS}}[H]$ is discussed in detail in [1]. The action (3.5) coincides for $n = 1$ with the topological mass term for the Abelian vector multiplet [7]. In the $n = 3$ case, (3.5) proves to be the linearised action for $\mathcal{N} = 1$ conformal supergravity [13, 5].

For $n = 1$ the field strength (3.1) is

$$W_\alpha = -\partial_\alpha{}^\beta H_\beta + \frac{i}{2} D^2 H_\alpha = i D^\beta D_\alpha H_\beta , \quad (3.6)$$

as a consequence of the anti-commutation relation

$$\{D_\alpha, D_\beta\} = 2i\partial_{\alpha\beta} . \quad (3.7)$$

The final expression for W_α in (3.6) coincides with the gauge-invariant field strength of a vector multiplet [13]. The Bianchi identity $D^\alpha W_\alpha = 0$ is a corollary of

$$D^\alpha D_\beta D_\alpha = 0 \quad \implies \quad [D_\alpha D_\beta, D_\gamma D_\delta] = 0 . \quad (3.8)$$

For $n = 2$ the field strength (3.1) can be seen to coincide with the gravitino field strength [13]. Finally, for $n = 3$ the field strength (3.1) is the linearised version [28] of the $\mathcal{N} = 1$ super-Cotton tensor [5, 6]. This is why (3.1) can be called the higher spin super-Cotton tensor.

It should be pointed out that (3.1) is the general solution of the constraint (3.4). The simplest way to prove this is the observation that the field strength (3.1) may be recast in the form

$$W_{\alpha(n)} \propto i^n D^{\beta_1} D_{\alpha_1} \dots D^{\beta_n} D_{\alpha_n} H_{\beta_1 \dots \beta_n} . \quad (3.9)$$

It is completely symmetric, $W_{\alpha_1 \dots \alpha_n} = W_{(\alpha_1 \dots \alpha_n)}$, as a consequence of (3.8).

Let us consider a higher-derivative $\mathcal{N} = 1$ superconformal theory with action

$$S[W] = i \int d^3 x d^2 \theta \varphi^4 L \left(\frac{W_{\alpha(n)}}{\varphi^{n+2}} \right) , \quad (3.10)$$

where φ is a real conformal compensator of dimension 1/2. This model possesses a dual description. Indeed, we can associate with (3.10) the following first-order model

$$S[\mathcal{W}, G] = i \int d^3 x d^2 \theta \left\{ \varphi^4 L \left(\frac{\mathcal{W}_{\alpha(n)}}{\varphi^{n+2}} \right) - i^n G^{\alpha(n)} \mathcal{W}_{\alpha(n)} \right\} , \quad (3.11)$$

where $\mathcal{W}_{\alpha(n)}$ is an unconstrained superfield, and the Lagrange multiplier has the form

$$G_{\alpha(n)} = i^n D_{(\alpha_1} \Psi_{\alpha_2 \dots \alpha_n)} , \quad \bar{\Psi}_{\alpha(n-1)} = \Psi_{\alpha(n-1)} . \quad (3.12)$$

Varying $S[\mathcal{W}, G]$ with respect to the Lagrange $\Psi_{\alpha(n-1)}$ leads us back to the original action (3.10), and therefore the models (3.10) and (3.11) are equivalent. On the other hand, we can integrate out the auxiliary superfield $\mathcal{W}_{\alpha(n)}$ from (3.11), which leads us to a dual action of the form

$$S[G] = \text{i} \int \text{d}^3x \text{d}^2\theta \varphi^4 L_{\text{D}}(\varphi^{n-2} G_{\alpha(n)}) . \quad (3.13)$$

Unlike the original action (3.10), which is a higher-derivative theory for $n > 1$, its dual $S_{\text{D}}[G]$ is free of higher derivatives. In the $n = 1$ case, the duality transformation described corresponds to the standard duality between the $\mathcal{N} = 1$ scalar and vector multiplets in three dimensions [9].

4 $\mathcal{N} = 0$ duality

The $\mathcal{N} = 1$ higher spin super-Cotton tensor and the $\mathcal{N} = 1$ duality transformation described in section 3 were obtained by performing the $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ superspace reduction, in analogy with the earlier results for extended supersymmetric nonlinear sigma models [29, 30]. Actually one can continue this process one step further and carry out the $\mathcal{N} = 1 \rightarrow \mathcal{N} = 0$ reduction. This gives (up to a factor) the higher-spin conformal field strength

$$C_{\alpha(n)}(h) := \sum_{J=0}^{\lfloor n/2 \rfloor} \binom{n}{2J+1} \square^J \partial_{(\alpha_1}^{\beta_1} \dots \partial_{\alpha_{n-2J-1}}^{\beta_{n-2J-1}} h_{\alpha_{n-2J} \dots \alpha_n) \beta_1 \dots \beta_{n-2J-1}} , \quad (4.1)$$

which is defined for $n \geq 2$. It is a descendant of the real prepotential $h_{\alpha(n)}(x)$ defined modulo gauge transformations of the form

$$\delta h_{\alpha(n)} = \partial_{(\alpha_1 \alpha_2} \zeta_{\alpha_3 \dots \alpha_n)} . \quad (4.2)$$

The field strength (4.1) is invariant under these gauge transformations,

$$\delta C_{\alpha(n)} = 0 , \quad (4.3)$$

and obeys the Bianchi identity

$$\partial^{\beta\gamma} C_{\beta\gamma\alpha_1 \dots \alpha_{n-2}} = 0 . \quad (4.4)$$

It may be shown that $C_{\alpha(n)}$ is a primary field of dimension $(1 + n/2)$ if the prepotential $h_{\alpha(n)}$ is primary of dimension $(2 - n/2)$. Associated with $C_{\alpha(n)}$ is the conformal Chern-Simons action [31]

$$S_{\text{CS}}[h] = \text{i}^n \int \text{d}^3x h^{\alpha(n)} C_{\alpha(n)}(h) , \quad (4.5)$$

which is invariant under the gauge transformations (4.2).

In the case of even rank, $n = 2s$, with $s = 1, 2, \dots$, the field strength (4.1) coincides with the bosonic higher spin Cotton tensor given originally by Pope and Townsend [31]. It reduces to the linearised Cotton tensor for $n = 4$, and to the Maxwell field strength for $n = 2$. It should be pointed out that the conformal spin-3 case, $n = 6$, was studied for the first time in [32]. In the case of odd rank, $n = 2s + 1$, eq. (4.1) describes fermionic higher spin conformal field strengths. They did not appear in [31]. The spin-3/2 case, $n = 3$, was considered in [33], where the field strength $C_{\alpha(3)}$ was called the Cottino tensor.

The field strength (4.1) proves to be the general solution to the conservation equation (4.4). This result has recently been proved in [34] in the bosonic case, $n = 2s$, and the proof given is quite nontrivial. There is an alternative proof based on supersymmetry considerations. The point is that the higher spin Cotton tensor $C_{\alpha(n)}$ may be imbedded into the $\mathcal{N} = 1$ super-Cotton tensor $W_{\alpha(n)}$ as its lowest (θ -independent) component. The latter obeys the constraint (3.4), which has the general solution (3.1) or, equivalently, (3.9). The fact that (3.9) is the general solution to (3.4), is a corollary of the $\mathcal{N} = 1$ identities (3.8).

Let us turn to introducing the aforementioned $\mathcal{N} = 0$ duality transformation. We consider a higher-derivative conformal theory with action

$$S[C] = \int d^3x \varphi^6 L \left(\frac{C_{\alpha(n)}}{\varphi^{n+2}} \right) , \quad (4.6)$$

where φ is a real conformal compensator of dimension 1/2. To obtain a dual description for the model, we can associate with (4.6) the following first-order model

$$S[\mathcal{C}, G] = \int d^3x \left\{ \varphi^6 L \left(\frac{\mathcal{C}_{\alpha(n)}}{\varphi^{n+2}} \right) - i^n G^{\alpha(n)} \mathcal{C}_{\alpha(n)} \right\} , \quad (4.7)$$

where $\mathcal{C}_{\alpha(n)}$ is an unconstrained field, and the Lagrange multiplier is

$$G_{\alpha(n)} = \partial_{(\alpha_1 \alpha_2} \Psi_{\alpha_3 \dots \alpha_n)} . \quad (4.8)$$

Varying $S[\mathcal{C}, G]$ with respect to $\Psi_{\alpha(n-2)}$ implies $\mathcal{C}_{\alpha(n)} = C_{\alpha(n)}$, and then $S[\mathcal{C}, G]$ reduces to the original action (4.6). On the other hand, we can first integrate out the auxiliary field $\mathcal{C}_{\alpha(n)}$ from (4.7), which leads to a dual action of the form

$$S[G] = \int d^3x \varphi^6 L \left(\varphi^{n-4} G_{\alpha(n)} \right) . \quad (4.9)$$

In the $n = 2$ case, the duality transformation described corresponds to the standard vector-scalar duality in three dimensions.

Acknowledgements:

I am grateful to Mirian Tsulaia for collaboration at the early stage of this project, Jim Gates for important correspondence, Ulf Lindström for encouragement, Joseph Novak for comments on the manuscript, and Paul Townsend for informing me of the Cottino tensor construction given in [33]. It is my pleasure to thank the Galileo Galilei Institute for Theoretical Physics for the hospitality and the INFN for partial support during the completion of the revised version of this work. This work is supported in part by the Australian Research Council, project No. DP160103633.

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